Subject 5. Probabilities and triangles

In the last year Math.en.Jeans workshop, our team struggled to solve the following two problems:

1. Assume a stick is randomly broken into three pieces. What is the probability that the obtained pieces can form a triangle?

2. Let be the set $A_n = \{1, 2, 3, 4, ..., n\}$. We randomly choose 3 of its elements. What is the probability that the chosen numbers be the lengths of the sides of a triangle?

Our new problems are:

1'. Assume a stick is randomly broken into four pieces. What is the probability that the obtained pieces can form a quadrilateral?

2'. Let be the set $B_n = \{1^2, 2^2, ..., n^2\}$. We randomly choose 3 of its elements. What is the probability that the chosen numbers be the lengths of the sides of a triangle?

Solve the same problem for the set $C_n = \{1^3, 2^3, ..., n^3\}$, or more general, for the set $X_{nk} = \{1^k, 2^k, ..., n^k\}$.

3. What is the probability that four randomly chosen numbers of the above sets of number (i. e. A_n , B_n , C_n , X_{nk}) be the lengths of the sides of a quadrilateral?