Problems 2019-2020
Souillac-Varsovie

EUCLIDEAN, SPHERIQUE AND HYPERBOLIC TILLING.

The plane, the sphere and the Poincaré half plane (or hyperbolic) represent in some sense the three alternatives involved in the fifth demand of Euclid: *given a straight line and a point, one can have, through this point*

- *a unique parallel straight line (Euclidean space)*
- *or, an infinite number of parallel straight lines (Hyperbolic space)*
- *or, no parallel straight line at all (Spheric space)*

The goal of the subject is to study the tilling of these spaces by polygons and see how their respective geometries have an influence on the existence of tilling.

In this project, *tilling* will mean the following. Consider a polygon P and all its transformation by isometries (in the plane, that would mean rotations, translations and axial symetries). Then P is said to give a tilling of the space if and only if you can cover the whole space with the pieces obtain by the above transformations starting from the sole polygon P.

Below, you'll find questions that can associate to the problem.

(1) Understand the introductive paragraph. (but If you have followed my talk at the congress last year, it will be easy)
(2) Consider a given triangle T in the plane. On what conditions on T, T gives a tilling of the plane.
(3) Same question as (2) for quadriangle.
(4) Same question as (2) for regular pentagon, hexagon, heptagon and so on.
(5) What is a straight line in the half hyperbolic plane ? Draw some triangle in the hyperbolic halp plane ?
(6) What are the isometries of the hyperbolic halp plane ?
(7) Write a program that draw a tilling of the hyperbolic half plane by a given regular n-gone.
(8) ....